Name: _____ Date: _____

PRACTICE 112m = 4 sec 6.1

Pythagorean Theorem

READ

When you know the x- and y- components of a vector, you can find its magnitude using the Pythagorean theorem. This useful theorem states that $a^2 + b^2 = c^2$, where a, b, and c are the lengths of the sides of any right triangle.

For example, suppose you need to know the distance represented by the displacement vector (4,3)m. If you walked east 4 meters then north 3 meters, you would walk a total of 7 meters. This is a distance, but it is not the distance specified by the vector, which describes the shortest way to go. The vector (4,3)m describes a single straight line. The length of the line is 5 meters because $4^2 + 3^2 = 5^2$.

The Pythagorean theorem can be used to help us calculate the magnitude of a vector once we know its components along the x- and y- directions. Also, we can find one of the components of the vector if we know the other component and the magnitude of the vector.

EXAMPLE

A displacement vector $\vec{x} = (2,3)$m has these components:

2 meters in the x direction.

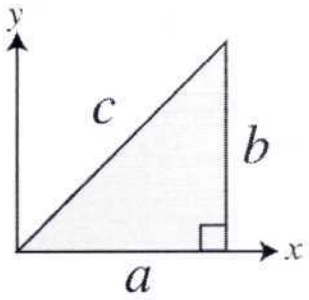
3 meters in the y direction.

The Pythagorean theorem

$$a^2 + b^2 = c^2$$

a and b are the lengths of the short sides of a right triangle.

c is the length of the side opposite the right angle.



What is the magnitude of the vector?

Using the Pythagorean theorem, a is the component along the x direction and b is the component along the y direction. The magnitude of the vector is c. We can find the magnitude by taking the square root of $a^2 + b^2$:

$$\sqrt{a^2 + b^2} = \sqrt{c^2}$$

$$\sqrt{(2\text{ m})^2 + (3\text{ m})^2} = \sqrt{c^2}$$

$$\sqrt{4\text{ m}^2 + 9\text{ m}^2} = \sqrt{c^2}$$

$$\sqrt{13\text{ m}^2} = 3.6\text{ m} = c$$

PRACTICE

1. Find the magnitude of the vector $\vec{a} = (3, 4)$.
2. Find the magnitude of the vector $= (-3, -4)$.
3. Find the magnitude of the vector $\vec{z} = (5, 0)$.
4. Find the magnitude of the vector $\vec{x} = (12.00, 6.00)$cm.

5. A robot starts from a certain point and moves east for a distance of 5.0 meters, then goes north for 3.0 meters, and then turns west for 2.0 meters.
   a. What are the x-y coordinates for the resultant vector?
   b. What is the magnitude of the resultant vector for the robot?

Challenge problems

6. Express the resultant vector in problem 5 above in polar coordinates (magnitude, angle). Assume that the positive x direction is from west to east and the positive y direction is from south to north.

7. A resultant vector has a magnitude of 25 meters. Its y component is –12 meters. What are its two possible x components?

8. For these vectors: $\vec{v}_1 = (5,0)$, $\vec{v}_2 = (0,-3)$, and $\vec{v}_3 = (1,0)$
   a. Add the vectors.
   b. Find the magnitude of the resultant vector.

9. For these vectors: $\vec{v}_1 = (-5,0)$, $\vec{v}_2 = (0,-2)$, and $\vec{v}_3 = (7,0)$
   a. Add the vectors.
   b. Find the magnitude of the resultant vector.

10. For these vectors: $\vec{v}_1 = (5,0)$, $\vec{v}_2 = (0,-5)$, and $\vec{v}_3 = (5,180°)$
    a. Add the vectors.
    b. Find the magnitude of the resultant vector.

11. For these vectors: $\vec{v}_1 = (5,45°)$, $\vec{v}_2 = (0,-10)$, and $\vec{v}_3 = (1,180°)$
    a. Add the vectors.
    b. Find the magnitude of the resultant vector.